Appetizer How unlucky is	unlucky?
	· · · · · · · · · · · · · · · ·
player lottery game company	
virtual item as reward or nothing	
Rules $\begin{cases} \$1 & per shot non-refundable \\ prob = \frac{1}{10} to win. \\ ployer way short as wrany times, way stop any time. \end{cases}$	
Outcome ? 30 shots total 1 win, 29 lose => observed prob = player sues the company	$\frac{1}{30}$
If you were the { player { , what company }	uould you do?
$P(1win 29 lose   P=\overline{ro}) = \binom{29}{i} \cdot (\overline{ro})$	$\frac{1}{0}\left(\frac{9}{10}\right)^{29} \gtrsim 0.137$
Is this enough to prove the com or the player is having bad luck?	pany gu>lty ?

P-value	• •
It's used to fost the null hypothesis Ho ( the initial humphace)	
If p(.05, then we reject Ho	• •
Otherwise we don't reject Ho	
Viscrete case in the second second second second	
p-value = the prob. of seeing sth that's equally rare	
+ the prob of seeing sth rarer or more extreme	• •
+ the prob of seeing the observed	• •
The for the large week Ho	• •
The Prod Carculated with the	
· · · / · · · · · · · · · · · · · · · ·	• •
Starting for Thing I Marine Ford	
Mo = Tair Com Tor > Times is the com Tair.	
result = 4 heads, 1 tail	• •
result = 4 heads, 1 tail $P(5H) = P(5T) = \frac{1}{22}$	· ·
result = 4 heads, 1 tail $P(4H,  T) = \frac{5}{32}$ $P(5H) = P(5T) = \frac{1}{32}$	· ·
$\begin{aligned} &\text{result} = 4 \text{ heads, }   \text{ tai}   \\ &P(4H,  T) = \frac{5}{32}  P(5H) = P(5T) = \frac{1}{32} \\ &P(4T,  H) = \frac{5}{32}  \text{; } P - \text{value} = \frac{5}{32} + \frac{5}{32} + \frac{1}{32} + $	
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Continuou (ase p-value = the blue area Ho its counter part the observed result If we reject Ho, then it means there is a better distribution to fit the data Otherwise Ho is good enough Approximate 5% of starts tests from the same distribution will be false positive remelor sample co calculate p-vale p=0,68,0,49,0,92,002 Ho How tall is human? \_\_\_\_\_\_Ho I knew they're around 5-7ft 光尖尖 What ru? Unfortunately, the alien went to a kindergarten, and recorded avg = 2.6 ft, SD = 0.4=) p = 0.005 => reject Ho it's a false positive . Huwan are shorter than I thought

Conti	usio	n wo	ITY IX	· · · · · · · · · · · · · · · · · · ·
		Reject null	hypothesis	
•		No P 70,05	<b>Yes</b> P< 0, 05	
Null hypothesis	True	True negative	False positive type   enor	$H_0 \sim healthy, Test \sim virus testpositive = reject H_0 = unhealthy= test positive$
· · · · · · · · · · · · · · · · · · ·	False	False negative -type 2 error	True positive	
specifici	$f_{\lambda}^{(1)} = \frac{1}{2}$	<u>TN</u> TN+FP		· · · · · · · · · · · · · · · · · · ·
Sensitivity	/ = Yec	$all = \frac{T}{TP}$	<u>P</u> + FN	
. A histogr	am of 10,000 p- taken from	values generated by testing the same distribution.	g samples	A histogram of 10,000 p-values generated by testing samples taken from two different distributions.
Number of p-values 8	fa 510 p	( <u>se</u> <u>positive</u> values (5.1%) are < 0.05	Number	of p-values
	0.0 0.2 Possible	0.4 0.6 0.8 values for p-values	1.0 hegative Now let's la	0.0 0.2 0.4 0.6 0.8 1.0 Possible values for p-values
	· · ·			p-value = 0.03
			Test #2	p-value = 0.01
p-values	are uni	form if draw		P-values are skewed or close to
from	the same	2 districe		zero if draw from diff. distri
· · · · · · · · ·	· · · ·	· · · · · · · ·	· · · · · · ·	
				· · · · · · · · · · · · · · · · · · ·

Likelihood
$L(\theta_{o};x) = Prob(X=x \theta=\theta) = f_{X}(x;\theta_{o})$
$L(\theta) = L(\theta; x) = \prod_{i=1}^{n} f_i(x_i; \theta)$
$x = $ the data, $\theta = $ the parameters, $n = $ # of samples
SCOVE folio)
$\log  i  kehood l(0) = \log L(0)$
Score equation for l(0) =0 is to find the wax l(0)
Observed information $I_0(\theta) = -\frac{d}{d\theta^2} l(\theta)$ quantifies the confidence
in the MLE (max likelihood estimation Expected information I(0) = F5 to (A:X)?
Expected value
Variance $V(0) \approx I(0)$
If O is multi-dim, O, then Score equ: SO, LLO) =0
(n + 0)  (n + 1)  (0
Bayes Thus
inkehood, c prior
$P(P D) = \frac{P(D P)P(P)}{P(D)}$
D= data D= data
$\theta = wooden on performance.$

eg flip a coin proportion of heads Prior 0.2 0.15 (H) 0.1 0.05  $P(\theta)$ 0 0.1 0 02 0.3 0.4 0.5 0.6 A let n=1, head=1 Likelihood 1 0.8 P(D)) 0.6 (Data | 0) 0.4 0.2 \*(not a pdf) 08 0 03 04 0.5 06 07 0 9 Posterior 0.2 0.15 PLOD 0.1 b(θ|Data) 0.05 0.1 0.6 0.7 0.5 θ If posterior has the same parametric form as prior Then the prior is called conjugate prior P(D(0) P(0)  $P(\theta(D) =$ SP(DID) P(D) do

Test (Suppose of is known)  $H_0: \mu = \mu_0,$ Test stat:  $Z = \frac{\overline{x} - \mu_0}{\overline{x}}, \text{ where } \overline{\nabla_{\overline{x}}} = \overline{\int_{\overline{n}}},$ E known If Ho is true, then Z will be in normal distri If |Z|Z 1.96, then reject Ho given \$2=0.05 Reject H<sub>0</sub> 0.025 Reject H<sub>0</sub> 0.025 We don't calculate p-value We convert a to the threshold Z value threshold use ppt to find H. UCU H,: 11>100 Reject H<sub>0</sub> 0.05  $\left| \begin{array}{c} D_{0} \ Not \\ Reject \ H_{0} \end{array} \right|$  Reject  $H_{0}$ 0.05 test (suppose t is unknown) Ho: M= Mo observed Test Stat:  $t = \frac{\overline{x} - \mu_0}{SE}$ , where  $SE = \frac{S}{Jn}$ , S = SD of xIf Ho is true, then t will be in t-distri with n-1 df

Chi-Squard test · goodness of fit · test for indep degree of freedom df = (r-1). (c-1), r=# of rons, c=# of columns  $\chi^2 = \sum \frac{(O_1 - e_1)^2}{e_1}$ ,  $O_1 = observed$ ,  $e_1 = expected$  (calcubred by Ho) Reject Ho if x2 > chi2.ppf(0.95, df) = 3.84 or if  $p = 1 - chi2.cdf(x^2, df) < 0.05$ PDF  $data \rightarrow x^2 \rightarrow p - value - compare$  p - value = threshold - 1area = p-value reject Ho or not The calculated x2 p-value threshold  $\rightarrow x^2$  threshold compare data  $\rightarrow x^2$ . PDF 5% area reject Ho or not

Interence for one variance Let T be SD of the population  $H_0$ :  $T = T_0$ S be SD of the samples test stat:  $x^2 = \frac{(n-1)s^2}{T_0}$ s' is unbiased estimator of or, ie. E(s')=02  $\frac{(n-1)s^2}{r^2}$  has  $x^2$ -distri with dt = n-1 (if the population is gaussian) eg. x = 0.05 A  $(1 - \alpha)100\%$  confidence interval for  $\sigma^2$ Calculate the  $(rac{(n-1)s^2}{\chi^2_{lpha/2}}, rac{(n-1)s^2}{\chi^2_{1-lpha/2}})$ confidence interval  $P(X_{1-\frac{n}{2}}^{2} < \frac{(n-1)s^{2}}{1+2} < x_{0}^{2}) = |-\infty|$  $1-\alpha$ =>  $P\left(\frac{(n-1)s^{2}}{x_{d}^{2}} < \sqrt{2} < \frac{(n-1)s^{2}}{x_{d}^{2}}\right) = 1 - \alpha$ Var use ppt to lower bound upper bound